ON THE DETERMINATION OF DISPLACEMENTS IN THE GALIN PROBLEM

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In the note [1], the solution of Galin [2] for the state of stress of a plane with a circular hole for biaxial extension (plane strain) has been achieved by construction of the displacement field.

In the case under consideration there occur constrained deformations; therefore, for the determination of the displacements in the plastic region one has to use the relations of the Prandtl-Reuss theory

$$\frac{2de_\rho{}^p}{\sigma_\rho - \sigma_\theta} = \frac{2de_\theta{}^p}{\sigma_\theta - \sigma_\rho} = \frac{de_{\rho\theta}{}^p}{2\tau_{\rho\theta}} \qquad (1)$$

Here the superscript p denotes that the plastic component of the deformation is considered.

Using the assumption of an incompressible material, one obtains the first equation for the determination of the displacements

$$e_\rho + e_\theta = 0 \qquad (2)$$

The second equation is obtained in the following manner. In the Galin problem, in the plastic retion, $\tau_{\rho\theta} = 0$; further

$$e_{\rho\theta} = e_{\rho\theta}{}^p + e_{\rho\theta}{}^e = e_{\rho\theta}{}^p + \frac{\tau_{\rho\theta}}{2G} \qquad (3)$$

where G is the shear modulus; the superscript e refers to the elastic part of the deformation in the plastic region; therefore, from (1) and (3) for the plastic region one obtains

$$de_{\rho\theta} = 0 \qquad (4)$$

One of the possible treatments of the Prandtl-Reuss theory consists of the utilization of the Eulerian representation of the displacement velocity field [3].

It will be shown that in the case under consideration the solution of the original equations for the Langrangian representation coincide with the solution in the Eulerian representation [1].

In fact, in this case the integration of (4) gives

$$e_{\rho\theta} = f(x,\ y,\ \lambda) \tag{5}$$

where λ is a load parameter.

However, on the strength of the character of the assumption underlying the Galin solution, the function on the right-hand side of (5) vanishes. This circumstance will now be proved. By the basic assumptions underlying the Galin theory the boundary of the plastic region cannot intersect the boundary of the hole. Thus, the character of the assumptions of Galin imposes an essential limitation on the loading process.

Consider the process of deformation. At the instant when a plastic region forms, the contour of the plastic region coincides with the edge of the hole. Consequently, at this instant an axisymmetric state of stress must prevail. For axisymmetric deformation one must have $e_{\rho\theta} = 0$ everywhere. Further, for the boundary of the elastic and plastic regions $\tau'_{\rho\theta} = 0$, and therefore it follows from Hooke's law that there $e_{\rho\theta} = 0$. Hence, for the subsequent expansion of the plastic region, at any fixed point at the instant of the passage through it of the boundary of the plastic region, one must have $e_{\rho\theta} = 0$. Further, by (4), one must have at any point of the plastic region $de_{\rho\theta} = 0$, and therefore, the strain $e_{\rho\theta}$ in the plastic region will always be zero, i.e. $f(x,\ y,\ \lambda) = 0$.

Thus, the results of the note [1] remain completely in force also in the case of the Lagrangean treatment of the relations of the theory of Prandtl-Reuss.

Some attention will now be given to the limitations imposed on the loading. In order to avoid loading, inadmissible for the loading conditions of the Galin problem, the plastic zone at any instant of loading must completely contain the plastic zone of any preceding instant of loading.

In the Galin problem the forces at infinity will be denoted by $A(\lambda)$ and $B(\lambda)$, the radius of the edge of the hole by R, the pressure at the contour of the hole by q. The boundaries of the plastic region are an ellipse with semi-axes $a(1 + \delta)$ and $a(1 - \delta)$ and

$$a = R \exp\left\{\frac{1}{2k}\left(\frac{A+B}{2} + q - k\right)\right\}, \qquad \delta = \frac{B-A}{2k} \qquad (B \geqslant A) \tag{6}$$

where k is the constant on the right-hand side of the plasticity condition.

Let the quantities at the preceding instant of loading be provided with a subscript 1, those at the following instant with the subscript 2.

Then one must have

$$a_2(1+\delta_2) \geqslant a_1(1+\delta_1), \qquad a_2(1-\delta_2) \geqslant a_1(1-\delta_1) \tag{7}$$

It follows immediately from (7) that $a_2 \geqslant a_1$. Using (6) and (7), one obtains the unknown limits of variation of the forces A, B, q. It is easily verified that there exists a region of variation of the loading.

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